Welcome back to Cuemath Valley!

Last month we learned that the number of COVID-19 cases is growing *exponentially* because the *virus reproduction number* R_0 is greater than 1.

This month we are exploring strategies to bring R_0 below 1, which would not only end exponential growth, but also bring the pandemic to an end!

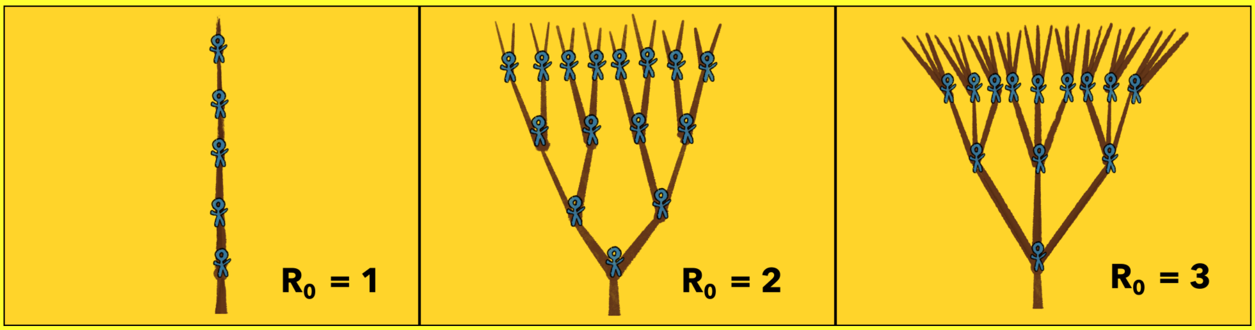
In this document for parents and educators, we will summarize the key takeaways from each section, and also offer suggestions for further exploration for you and your child or student.

SECTION 1:

In Section 1 of this month's activity, we reviewed *infection trees* and the *virus reproduction number* R_0.

Remember R_0 is defined to be the *average number of new people who contract the virus from an infected person*.

And *infection trees* provide a very nice way to visualize R_0. In particular, infection trees show just how drastically the growth rate of the infection changes based on a small change in R_0.



It is important to note that R_0 is not fixed, but varies based on the changes we make and the precautions we follow.

As a <u>further exploration</u>, you can try looking up the current value of R_0 in different parts of the world, and perhaps where you live. Here are some links to help you get started:

http://metrics.covid19-analysis.org/

https://rt.live/

SECTION 2:

In Section 2 we asked how we can go about increasing or decreasing R_0. The key idea is that we can break R_0 down as the product of two factors:

$R_0 = A \times B$, where

A = *the average number of other people an infected person has close encounters with while they are contagious*, and

B = *the average chance of infection during any close encounter*.

Since R_0 is the product of A and B, it follows that R_0 is *directly proportional* to both A and B. Thus, multiplying either A or B by any number will multiply R_0 by that same number!

This gives us two strategies for reducing R_0:

1. We can reduce A, the number of people an infected person meets, via *social distancing*.

2. Or we can reduce B, the chance of infection during any meeting, using widespread *face masks*.

As further exploration, you can learn about how these strategies pan out with other epidemics. R_0 is also directly proportional to the *number of days an infected person is contagious*. So a third strategy is to use medication to reduce that number. In fact, this is exactly the strategy many anti-retroviral drugs use to reduce R_0 in the case of HIV. If you're interested, here's a good document to begin your exploration with:

https://web.stanford.edu/~jhj1/teachingdocs/Jones-Epidemics050308.pdf

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SECTION 3:

Section 3 explored *social distancing* as a way to reduce R_0, by reducing the number of people an infected person comes into close contact with.

In offices or classrooms, social distancing becomes a geometry problem of efficiently arranging desks so that we can fit as many people as possible while still maintaining a safe distance.

In other words, it becomes a circle-packing problem! Mathematicians proved in the 1940's that the *hexagonal packing* used by honeybees is the most efficient circle packing, with the greatest *packing density*.

But that's only the beginning of the story! As further exploration, you can learn about circle packing in higher dimensions. In three dimensions the problem becomes one of *sphere packing*. For example, what is the most efficient way to pack tennis balls, or oranges. It may sound simple, but the answers are surprising.

Here is a nice article to get you started, relating circle packing to social distancing but also exploring the higher dimensional analogues.

https://www.quantamagazine.org/the-math-of-social-distancing-is-a-lesson-in-geometry-20200713

SECTION 4:

In the real world, social distancing alone can only reduce R_0 so much. Humans are social creatures after all! But with face masks we can dramatically reduce the chance of infection in any meeting.

In fact, face masks are much more effective than you might have guessed. If a simple cloth face mask is worn properly, it blocks out 60% of the viruses that try to pass through it, and hence allows only 40% through.

But face masks filter viruses on both inhalation and exhalation!
So only 16% of the viruses will make it from one person to another, provided they are both wearing 60% effective masks, since 40% of 40% is 16%. If two people are both wearing face masks, the impact squares!

For further exploration, you can try taking a more hands on approach to the probabilities discussed so far, and in doing so also explore the *Law of Large Numbers*, which has been secretly at play.

Here are two activities to get you started:

1. **Get out a coin and imagine a mask with 50% effectiveness, so each time a virus tries to pass through, there is a 50% chance it gets stuck, and a 50% chance it succeeds. Suppose 10 viruses are trying to get through the mask. Let's do an experiment: flip the coin 10 times to see how many of the 10 viruses succeed.**

   You can't predict the outcome of the experiment without actually doing it! Our intuition tells us 5 is the most likely number, but if you actually try the experiment, you may get a different number! So here is your task: do the experiment 10 times and record the result of each experiment (this will involve a total of 100 coin flips). Now, average your ten numbers.

   You've just seen the Law of Large Numbers at work, since the average will be closer to the expected value of 5. If you were to do 100 experiments and average the results, you would get even closer still! And that's a hint of why, when we're dealing with 100 million viruses, we can assume half of them get through a mask with 50% effectiveness.

2. **Now let's try to understand why with two masks the effectiveness squares. Imagine two masks, each with 50% effectiveness, and find two coins, one for each mask. Each time a virus attempts to pass through both masks, it must get a heads on each coin to succeed. If either coin comes out tails the virus will fail to pass through.**

   This time let's just imagine 10 viruses trying to pass through both masks. How many make it through? You'll have to flip both coins 10 times to find out. Because 10 is a small number, the answer may not be close to the expected number of 2.5. But this experiment helps give you a sense of why the probability squares, resulting in just a 25% chance that any given virus will make it through.

SECTION 5:

As Dr. Atul Gawande wrote in The New Yorker,

“ **if at least 60% of the population wore masks that were just 60% effective** in blocking viral transmission—which a well- fitting, two-layer cotton mask is —**the epidemic could be stopped.**”

In Section 5, we put everything together to create a very basic simulation of the pandemic, in order to see why Dr. Atul Gawande’s statement is true.

It turns out that if 60% of people wear a 60% effective mask, then the average chance of infection is cut to just 41% of what it would have been without masks. And since R_0 is directly proportional to the chance of infection, R_0 is also multiplied by 0.41.

This gives us a strategy for ending the pandemic. With social distancing we can bring R_0 down to 2, and then with 60% of people wearing 60% effective masks, we can bring R_0 down to 0.82.

As further exploration, you can look at how this model can be made more realistic, and also explore how changing the parameters of the model affects the outcome.

One simplifying assumption we made is that masks are equally effective on inhalation and exhalation. In reality the percent of viruses blocked on inhalation and exhalation may differ.

If you'd like to explore these ideas further, there is a very nice interactive version of this model in which you can adjust the parameters and see the effect on R_0:

https://aatishb.com/howmaskswork