PROBLEMS FROM DAY 3

AFS I: ALGEBRA

Problem 1. How many elements of order 5 are in a group of order 20?

Problem 2. (a) Prove that there are no simple groups of order pq, where p and q are both prime.
(b) Prove that there are no simple groups of order p^2q, where p and q are both prime.

Problem 3. Give an explicit description of each Sylow 2-subgroup in:
(a) D_{10} (the dihedral group of order 20),
(b) T (the tetrahedral group), and
(c) O (the octahedral group).

Problem 4. Classify all groups of order 33.

Problem 5. Let G be a group of order $n = p^e m$, where p is a prime that does not divide m. Prove that G has subgroups of order p^r for each $r \in [1, e]$.

Challenge 1. Prove that there is no injective homomorphism from the quaternion group Q_8 to the symmetric group S_7.

(Remember that Caley's Theorem shows we can always inject a group of order n into S_n. It turns out this is the best we can do in general, as this example demonstrates.)

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