

PROBLEMS FROM DAY 4

AFS I: ALGEBRA

Problem 1. Let $G = D_3 = \{1, x, x^2, y, xy, x^2y\}$. Consider the action of G on itself via left multiplication.

- (a) Let $H < G$ be the stabilizer of the subset $U = \{x, x^2y\}$. Calculate H .
- (b) Determine all the H -orbits in G .
- (c) Determine all the subsets of G that are stabilized by H .

Problem 2. Classify all groups of order $2p$, where p is prime.

Problem 3. (1) Let G be a group of order 96. Prove that G is not simple (i.e. prove that G must have a normal subgroup).

(2) Let G be a group of order 56. Prove that G is not simple.

Problem 4. Let p be a prime number. Find all the Sylow p -subgroups in the group $GL_n(\mathbb{F}_p)$.

Problem 5. Prove that there is no injective homomorphism from the quaternion group Q_8 to the symmetric group S_7 .