SCHUBERT POLYNOMIALS DAY 1

ATMW SCHUBERT VARIETIES 2017

Problem 1. For each permutation $w \in \{4612375, 6174235, 6324571\} \subset S_7$,

(a) Compute the inversion set $I(w)$, draw the Rothe diagram $D(w)$, write the Lehmer code $c(w)$, draw the shape $\lambda(w)$, and mark the values of the rank function on the essential set $\mathrm{Ess}(w)$.

(b) Verify that the permutation w can be reconstructed from its Lehmer code, as well as from the values of the rank function on the essential set.

(c) Give a reduced decomposition for w.

Problem 2. (a) Give an algorithm for producing a reduced decomposition for a permutation $w \in S_n$, and show that your algorithm does indeed produce a decomposition whose length is equal to $l(w)$ (i.e. the cardinality of the inversion set $I(w)$). Does your algorithm agree with either of the 'magic tricks' described in lecture?

(b) Show that any decomposition for w must have length at least $l(w)$. Hence the shortest possible decompositions for w have length exactly equal to $l(w)$.

Problem 3. Suppose $v \leq w$ in S_n, and let $v = s_{j_1} \dots s_{j_m}$ be a reduced decomposition. Can we always find a reduced decomposition for $w = s_{i_1} \dots s_{i_l}$ such that $(j_1, \dots, j_m)$ is a subsequence of $(i_1, \dots, i_l)$?

Problem 4. We define the *left* weak and strong Bruhat orders by using left multiplication to define our covering relations. In particular we say v precedes w in the left weak (respectively strong) Bruhat order if $l(v) + 1 = l(w)$ and $s_i v = w$ for some simple transposition s_i (respectively $t_{ij} v = w$ for some transposition t_{ij}).

(a) Show (without using the subword property) that the left strong Bruhat order coincides with the right strong Bruhat order defined in lecture.

(b) Does the left weak Bruhat order coincide with the right weak Bruhat order?

Problem 5 (Deletion Property)**.** Suppose $s_{i_1} \cdots s_{i_m}$ is a nonreduced decomposition of a permutation w. Show that there exist integers $p < q$ such that $w = s_{i_1} \cdots \hat{s_{i_p}} \cdots \hat{s_{i_q}} \cdots s_{i_l}$. Here you may want to use the exchange lemma from lecture.

Date: October 23, 2017.