SCHUBERT POLYNOMIALS DAY 2

ATMW SCHUBERT VARIETIES 2017

Problem 1 (Length of a permutation)**.** Pick a permutation $w \in S_4$ of length at least 4.

(a) Verify that permuting the columns of $I(w)$ by w^{-1} yields $D(w)$.

(b) Fix a reduced decomposition $w = s_{i_1} \cdots s_{i_l}$. Verify that $I(w) = \{s_{i_l} s_{i_{l-1}} \cdots s_{i_{h+1}}(i_h, i_h+1), 1 \leq h \leq l\}$.

Problem 2 (Bruhat Order Practice)**.** Pick permutations v and w in $S_4 \setminus \{e, w_0\}$ such that that $l(w) - l(v) = 1$, but such that $v \not\leq w$. Show that $v \not\vdash \leq w$ using the subword property, using key tableux, and using the rank functions.

Problem 3 (Notable permutations in S_4)**.** (a) Find all Grassmannian permutations in S_4. Calculate $D(w), c(w)$, and $\lambda(w)$ for each of them. Can you make a conjecture about properties these must satisfy?

(b) Find all 321-avoiding permutations in S_4. Draw $D(w)$ for each. Can you notice any pattern?

(c) Pick any 321-avoiding permutation w. Draw the graph $\mathcal{G}(w)$ of all reduced decompositions for w. Is there anything special about this graph?

Problem 4 (Nonreduced words)**.** Suppose $s_{i_1} \cdots s_{i_m}$ is a nonreduced decomposition of a permutation w.

(a) Deletion Property: Show that there exist integers $p < q$ such that $w = s_{i_1} \cdots \hat{s_{i_p}} \cdots \hat{s_{i_q}} \cdots s_{i_m}$. Here you may want to use the exchange lemma from lecture.

(b) Show that $s_{i_1} \cdots s_{i_m}$ is equivalent, via only braid relations and commutation relations, to a word $s_{j_1} \cdots s_{j_m}$, such that $s_{j_k} = s_{j_{k+1}}$ for some k.

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