SCHUBERT POLYNOMIALS DAY 3

ATMW SCHUBERT VARIETIES 2017

Problem 1 (Wiring Diagrams for Words)**.** (a) Show that a decomposition for a permutation w is reduced if and only if no two wires cross twice in its wiring diagram.

(b) Suppose no two wires cross twice in a wiring diagram for w. Show that wires i and j in the wiring diagram cross if and only if $(i, j) \in I(w)$.

(c) Can you use this result to give a quick proof of the Deletion Property? What about the Strong Exchange Property?

Problem 2 (Wiring Diagrams and Connectedness of Reduced Word Graph)**.** Let $w = s_{i_1} \cdots s_{i_l}$ be reduced, and let $m = \min\{i_1, \ldots, i_l\}$. Check what happens to the wiring diagram as you follow the algorithm from class for pushing s_m to the right as far as possible.

Problem 3 (Basic properties of divided difference operators)**.** (a) Show that for any polynomial $P \in \mathbb{Z}[x_1, \ldots, x_n]$, we have that $(x_i - x_{i+1})$ divides $P - s_i(P)$. Thus $\partial_i(P)$ is a well-defined polynomial for all $1 \leq i \leq n-1$.

(b) Show that if P is homogeneous of degree d, then $\partial_i(P)$ is either homegeneous of degree $d-1$, or zero.

(c) Show that $\partial_i(PQ) = (\partial_i P)Q + (s_i P)(\partial_i Q)$ for polynomials P and Q. Thus ∂_i behaves a bit like a derivative.

(d) Show that if $\partial_i(P) = 0$, then P is symmetric in x_i and x_{i+1}.

Problem 4 (Playing with the operators)**.** Let $P = x^2 y \in \mathbb{Z}[x, y, z]$. Find all polynomials we can generate using the operators ∂_1 and ∂_2. You can save some time by invoking the results of the next problem below!

Problem 5 (Relations satisfied by the operators)**.** Prove the following facts about the operators ∂_i on $\mathbb{Z}[x_1, \ldots, x_n]$.

(a) $\partial_i^2 = 0$,

(b) $\partial_i \partial_j = \partial_j \partial_i$ if $|i - j| > 1$, and

(c) $\partial_i \partial_j \partial_i = \partial_j \partial_i \partial_j$ if $|i - j| = 1$.

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