WORKSHEET 3

INTRODUCTION TO TOPOLOGY

Problem 1. Determine $\mathrm{Int}(A)$, $\mathrm{Cl}(A)$, and ∂A in each case.
(a) $A = (0, 1]$ in lower limit topology on $\mathbb{R}$.
(b) $A = \{a\}$ in $X = \{a, b, c\}$ with topology $\{X, \varnothing, \{a\}, \{a, b\}\}$.
(c) $A = \{a, c\}$ in $X = \{a, b, c\}$ with topology $\{X, \varnothing, \{a\}, \{a, b\}\}$.
(d) $A = (-1, 1) \cup \{2\}$ in the standard topology on $\mathbb{R}$.

Problem 2. Prove that $\mathrm{Cl}(\mathbb{Q}) = \mathbb{R}$ in the standard topology on $\mathbb{R}$.

Problem 3. Let X be a topological space, A a subset of X, and y an element of X. Prove that $y \in \mathrm{Cl}(A)$ if and only if every open set containing y intersects A.

Problem 4. In each case, determine whether the relation in the blank is $\subset$, $\supset$, or $=$. In cases where equality does not hold, provide an example indicating so.
(a) $\mathrm{Cl}(A) \cap \mathrm{Cl}(B)$ ____ $\mathrm{Cl}(A \cap B)$.
(b) $\mathrm{Cl}(A) \cup \mathrm{Cl}(B)$ ____ $\mathrm{Cl}(A \cup B)$.

Problem 5. Determine the set of limit points of
(a) the the interval $[0, 1]$ in the finite complement topology on $\mathbb{R}$.
(b) the set $A = \{\frac{1}{m} + \frac{1}{n} \in \mathbb{R} | m, n \in \mathbb{Z}_+\}$ in the standard topology on $\mathbb{R}$.
(c) the set $S = \{(x, \sin(\frac{1}{x})) \in \mathbb{R}^2 | 0 < x \leq 1\}$ as a subset of $\mathbb{R}^2$ in the standard topology.

Problem 6. Let A be a subset of $\mathbb{R}^2$ in the standard topology. Prove that if x is a limit point of A, then there is a sequence of points in A that converges to x.

Problem 7. Determine ∂A where $A = [0, 1]$ in the finite complement topology on $\mathbb{R}$.

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