QUIZ

INTRO TO REAL ANALYSIS

Problem 1. (a) Show that

$$\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots$$

converges and find the limit.

(b) Does

$$\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots$$

converge? If so, find the limit.

Problem 2. Give an example of each of the following, or argue that such a request is impossible.

(a) A sequence that has a subsequence that is bounded but contains no subsequence that converges.

(b) A sequence that does not contain 0 or 1 as a term but contains subsequences converging to each of these values.

(c) A sequence that contains subsequences converging to every point in the infinite set

$$\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\}.$$

(d) A sequence that contains subsequences converging to every point in the infinite set

$$\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\},$$

and no subsequences converging to points outside of this set.

Problem 3. Decide whether the following propositions are true or false, providing a short justification for each conclusion.

(a) If every proper subsequence of (x_n) converges, then (x_n) converges as well.

(b) If (x_n) contains a divergent subsequence, then (x_n) diverges.

(c) If (x_n) is bounded and diverges, then there exist two subsequences of (x_n) that converge to different limits.

(d) If (x_n) is monotone and contains a convergent subsequence, then (x_n) converges.