PROBLEM SET 1

ANALYSIS II

Problem 1. Consider $f(x) = 1/x$ over the interval $[1,4]$. Let P be the partition consisting of the points $\{1, \frac{3}{2}, 2, 4\}$.
(a) Compute $L(f,P), U(f,P)$, and $U(f,P) - L(f,P)$.
(b) What happens to the value of $U(f,P) - L(f,P)$ when we add the point 3 to the partition?
(c) Find a partition P' of $[1,4]$ for which $U(f,P') - L(f,P') < \frac{2}{5}$.

Problem 2. Suppose f_n is an integrable function on $[a,b]$ for each natural number n. If $(f_n) \to f$ uniformly on $[a,b]$, prove that f is also integrable on this set. (We will see that this conclusion can fail to hold if convergence is merely pointwise.)

Problem 3. Let $f : [a,b] \to \mathbb{R}$ be monotone increasing on the set $[a,b]$. Show that f is integrable on $[a,b]$.

*All questions taken from *Understanding Analysis: 2nd Edition* by Stephen Abbott.

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