PROBLEM SET 3

ANALYSIS II

Problem 1. Let V be an inner product space (over the reals).
(a) Prove the Cauchy-Schwarz inequality: $\langle \mathbf{x}, \mathbf{y} \rangle \leq ||x|| \cdot ||y||$.
(b) Prove $||\mathbf{x} + \mathbf{y}|| \leq ||x|| + ||y||$.
(c) Prove $||\mathbf{x} - \mathbf{y}|| \geq ||x|| - ||y||$.

Problem 2. Show that the sup norm on $\mathbb{R}^2$ is not derived from an inner product on $\mathbb{R}^2$.

Problem 3. Show that the the function

$$\langle \mathbf{x}, \mathbf{y} \rangle = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

is an inner product on $\mathbb{R}^2$.

Problem 4. Consider the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}.$$

(a) Find two different left inverses for A.
(b) Show that A has no right inverse.

Problem 5. Let A be an n by m matrix with $n \neq m$.
(a) If rank $A = m$, show there exists a matrix D that is a product of elementary matrices such that

$$D \cdot A = \begin{bmatrix} I_m \\ 0 \end{bmatrix}.$$

(b) Show that A has a left inverse if and only if rank $A = m$.
(c) Show that A has a right inverse if and only if rank $A = n$.

*All questions taken from *Analysis on Manifolds* by James Munkres.

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